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The Research of the Robust Stability in Linear System

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***Abstract* —The robust stability of control systems where the controlled plant possesses dynamics is relevant today. This paper focuses on robust stability analysis of system with conditions of Lyapunov functions. We propose a method for construct the Lyapunov function for linear system, and then we apply geometric meaning to investigate the region of stability. In this paper we received the Lyapunov function, geometric interpretation, gradient vector components and superstability condition of system. We made comparative analysis of examples and for all the examples the stability conditions of the system executed. This work presents some theoretical fundamental and practical results assisting in analyzing of the behavior of control systems, meaning of robust radius of stability. The general problem of robust stability is defined and conditions is given.**

***Keywords* — Stability, Linear systems, Robust control, Control theory, Lyapunov function.**

I. Introduction

The Robust analysis for linear systems is one of the actual direction today. Models with parametric uncertainty perform important function in both the theory and practical applications of robust control. They are described by the mathematical model containing parameters that are not precisely known, but the values are within given intervals. Such type of uncertainty can occur in the control of real processes, for example, as a result of modeling effort, inaccurate measuring (worn parts, weight change of the aircraft, temperature, fuel quality) or the influence of certain external conditions.

The high research interest of robust stability analysis techniques was developed before. Nevertheless, many of them specialized for concrete systems of uncertainty structure.

This paper provides a method aimed at usage of a universal approach in robust stability analysis for systems with parametric uncertainty. The present investigation method is based mainly on the combination of geometrical interpretation of Lyapunov function and the theory of robust stability of linear control systems, which is greatly beneficial especially for more complex tasks.

An important task is to solve the problem of analysis of control systems and synthesis of control laws. All this ensures the best protection from high uncertainty of object properties.

The considered problem is robust controllability of linear systems with parametric or non-parametric uncertainties [1,2]. Assuming that the linear system is controllable, a sufficient condition is proposed to preserve the properties of object (parameters of control systems) when system uncertainties are introduced. The most important idea in the study of robust stability is to specify constraints for changes in control system parameters that preserve stability.

The theory of robust control began in the late 1970s and early 1980s and soon developed a number of techniques for dealing with bounded system uncertainty. Today we see many works in this field [3-6].

For the purpose of studying the system dynamics and their control, we considered models of observing input and output signals of the object and the representing its behavior in the state space as most suitable.

This paper presents the approach of the construction of Lyapunov functions based on the geometric interpretation of the Lyapunov's direct method (also called the second method of Lyapunov) [7] and on gradient of dynamical systems in the state space of systems.

The content of this paper is organized in next way: In section 2, we introduce the basic equations of the model and their expanded form. In section 3 we received the Lyapunov function, geometric interpretation, gradient vector components and superstability condition of system. In section 4, we have considered the existence and robust stability, the radius of the robustness and considered a case study of the simulation practical example. Section 5 contains concluding remarks.

II. Mathematical Model Formulation

The control system is given by the linear equation.

$$\dot{x} = Ax + Bu, x \in R^n, u \in R^m \qquad (1)$$

$$y = Cx, y \in R^l$$

The controller is described by the equation

$$u = -Kx \qquad (2)$$

or $$u_i = -k_{i1}x_1 - k_{i2}x_2 - \ldots - k_{in}x_n, i = 1,2,\ldots,m$$

Description of parameters

$$A \in R^{n\times n},\ B \in R^{n\times m}, C \in R^{l\times n}, K \in R^{m\times n}$$

Matrices of the object, control, output and coefficients of control system, $x(t) \in R^n$ - state vector, $u(t) \in R^m$ - vector control, $y(t) \in R^l$ - vector output of the system.

We can provide equation (1) in expanded form:

$$\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + ... + b_{1m}u_m \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + ... + b_{2m}u_m \\
... & \qquad ... \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + ... + b_{nm}u_m
\end{aligned} \quad (3)$$

Let us denote $G = A - BK$ matrix of the closed system and the system (3) in matrix-vector form, we can write

$$\dot{x} = G\ x,\ x(t) \in R^n,$$
$$g_{ij} = a_{ij} - \sum_{k=1}^{m} b_{ik}k_{kj}$$

Therefore equation (3) can be written as

$$\begin{cases}
\dot{x}_1 = \left(a_{11} - \sum_{k=1}^{m} b_{1k}k_{k1}\right)x_1 + \left(a_{12} - \sum_{k=1}^{m} b_{1k}k_{k2}\right)x_2 + ... + \left(a_{1n} - \sum_{k=1}^{m} b_{1k}k_{kn}\right)x_n \\
\dot{x}_2 = \left(a_{21} - \sum_{k=1}^{m} b_{2k}k_{k1}\right)x_1 + \left(a_{22} - \sum_{k=1}^{m} b_{2k}k_{k2}\right)x_2 + ... + \left(a_{2n} - \sum_{k=1}^{m} b_{2k}k_{kn}\right)x_n \\
... \\
\dot{x}_n = \left(a_{n1} - \sum_{k=1}^{m} b_{nk}k_{k1}\right)x_1 + \left(a_{n2} - \sum_{k=1}^{m} b_{nk}k_{k2}\right)x_2 + ... + \left(a_{nn} - \sum_{k=1}^{m} b_{nk}k_{kn}\right)x_n
\end{cases} \quad (4)$$

III. The Geometric Approach of The Lyapunov Function

Stability is a fundamental notion in the qualitative theory of differential equations and is essential for many applications. In turn, Lyapunov functions are basic instrument for studying stability; however, there is no universal method for constructing Lyapunov functions. Nevertheless, in some special cases, a function can be constructed by applying special techniques. We construct the Lyapunov function for system and then use geometric interpretation to find the region of stability.

The direct method is a great advantage in the case of nonlinear systems. The method of constructing a Lyapunov function for stability determination is called the second method of Lyapunov. We use the «second method of Lyapunov» or the «direct method» as applied to linear systems.

Lyapunov's theorem has a simple geometric interpretation. The geometric meaning of a Lyapunov function used for determining the system stability around the zero equilibrium and can be used to solve the problem of constructing Lyapunov functions.

The geometric identification of stable states is reduced to creating a family of closed surfaces that surround the zero equilibrium of coordinates. The system state moves across contour curves: each integrated curve can cross each of these surfaces.

We suppose that there exists a positive definite function $V(x_1, x_2, ..., x_n)$ for which $(dV/dt < 0)$, and consider any integral curve of (3), coming out at the initial time of any point of the origin.

If dV/dt is a function with negative definite $(dV/dt < 0)$, then every integral curve starting from a sufficiently small neighborhood of the origin, will be sure to cross each of the surfaces $V(x_1(t), x_2(t),.., x_n(t)) = C, C = const$ of the outside to the inside, as the $V(x_1(t), x_2(t),.., x_n(t)) = C$ function is continuously decreasing.

But in this case the integral curves have to be infinitely close to the origin, i.e. unperturbed motion is asymptotically stable [2].

Thus, from the geometric interpretation point of view the second method of Lyapunov, the study of stability is reduced to the construction of a family of closed surfaces surrounding the origin. As the integral curves have property to intersect each of these surfaces, then stability of the unperturbed motion will be set [2].

Let us consider, that the expression $dV(x)/dt < 0$ means,that

$$\frac{dV(x)}{dt} = \frac{\partial V(x)}{\partial x}\frac{dx}{dt} = \left|grad\,V(x)\right|\left|\frac{dx}{dt}\right|\cos\alpha < 0,$$

i.e. scalar product of the gradient vector Lyapunov functions $grad\,V(x)$ by the velocity vector dx/dt for the asymptotic stability of the system must be less than zero.

This condition will be true if the angle α between the gradient of the Lyapunov function $grad\,V(x)$ and the velocity vector dx/dt forms an obtuse angle $90^0 < \alpha \le 180^0$.

The gradient vector of the Lyapunov function is always directed from the origin toward the highest growth of Lyapunov functions.

Also note that, in the study of stability [1] the origin corresponds to the stationary states of the system or the set of the system. The state equation (1) or (4) shall be made in respect to deviations from the steady state $X_s\,(x = \Delta x = X(t) - X_s(t))$.

Therefore the left side of (1) or (4), dx/dt expresses the velocity vector changes and deviations. We can assume that the velocity vector of deviations submitted to the stability of a system to the origin.

Components of the gradient vector Lyapunov functions in the opposite direction, but they are equal in absolute value. Then, if the Lyapunov function $V(x)$ is specified as a vector of functions $V(V_1(x), V_2(x),..., V_n(x))$ then gradient vector Lyapunov function can be written as $\partial V/\partial x = -dx/dt = -(A - BK)x.$ [7,8]

Vector components of the gradient of a potential function $V(x_1,...,x_n)$ are given in the form of vector Lyapunov functions with components $(V_1(x_1,x_2,...,x_n),V_2(x_1,x_2,...,x_n),...,V_n(x_1,x_2,...,x_n))$ we write in the form:

$$\begin{cases} -\dfrac{dx_1}{dt} = \dfrac{\partial V_1(x)}{\partial x_1} + \dfrac{\partial V_1(x)}{\partial x_2} + ... + \dfrac{\partial V_1(x)}{\partial x_n} \\ -\dfrac{dx_2}{dt} = \dfrac{\partial V_2(x)}{\partial x_1} + \dfrac{\partial V_2(x)}{\partial x_2} + ... + \dfrac{\partial V_2(x)}{\partial x_n} \\ .... \\ -\dfrac{dx_n}{dt} = \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} + ... + \dfrac{\partial V_n(x)}{\partial x_n} \end{cases} \quad (5)$$

In this system by substituting values of the components of the velocity vector we get:

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_1} + \dfrac{\partial V_1(x)}{\partial x_2} +,...,+ \dfrac{\partial V_1(x)}{\partial x_n} = \\ -\left(a_{11} - \sum_{k=1}^m b_{1k}k_{k1}\right)x_1 - \left(a_{12} - \sum_{k=1}^m b_{1k}k_{k2}\right)x_2 -,...,-\left(a_{1n} - \sum_{k=1}^m b_{1k}k_{kn}\right)x_n, \\ \dfrac{\partial V_2(x)}{\partial x_1} + \dfrac{\partial V_2(x)}{\partial x_2} +,...,+ \dfrac{\partial V_2(x)}{\partial x_n} = \\ -\left(a_{21} - \sum_{k=1}^m b_{2k}k_{k1}\right)x_1 - \left(a_{22} - \sum_{k=1}^m b_{2k}k_{k2}\right)x_2 -,...,-\left(a_{2n} - \sum_{k=1}^m b_{2k}k_{kn}\right)x_n \\ ... \\ ... \\ \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} +,...,+ \dfrac{\partial V_n(x)}{\partial x_n} = \\ -\left(a_{n1} - \sum_{k=1}^m b_{nk}k_{k1}\right)x_1 - \left(a_{n2} - \sum_{k=1}^m b_{nk}k_{k2}\right)x_2 -,...,-\left(a_{nn} - \sum_{k=1}^m b_{nk}k_{kn}\right)x_n \end{cases} \quad (6)$$

From here we can find the components of the gradient vector for the component vector functions

$$(V_1(x_1,x_2,...,x_n),V_2(x_1,x_2,...,x_n),...,V_n(x_1,x_2,...,x_n))$$

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_1} = -\left(a_{11} - \sum_{k=1}^m b_{1k}k_{k1}\right)x_1, \dfrac{\partial V_1(x)}{\partial x_2} = -\left(a_{12} - \sum_{k=1}^m b_{1k}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_1(x)}{\partial x_n} = -\left(a_{1n} - \sum_{k=1}^m b_{1k}k_{kn}\right)x_n \\ \dfrac{\partial V_2(x)}{\partial x_1} = -\left(a_{21} - \sum_{k=1}^m b_{2k}k_{k1}\right)x_1, \dfrac{\partial V_2(x)}{\partial x_2} = -\left(a_{22} - \sum_{k=1}^m b_{2k}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_2(x)}{\partial x_n} = -\left(a_{2n} - \sum_{k=1}^m b_{2k}k_{kn}\right)x_n \\ ... \\ \dfrac{\partial V_n(x)}{\partial x_1} = -\left(a_{n1} - \sum_{k=1}^m b_{nk}k_{k1}\right)x_1, \dfrac{\partial V_n(x)}{\partial x_2} = -\left(a_{n2} - \sum_{k=1}^m b_{nk}k_{k2}\right)x_2, \\ ..., \dfrac{\partial V_n(x)}{\partial x_n} = -\left(a_{nn} - \sum_{k=1}^m b_{nk}k_{kn}\right)x_n \end{cases} \quad (7)$$

Total time derivative of the components of the vector Lyapunov function $V_i(x)$ given by the equation of motion (1) and (4) is determined by

$$\frac{dV_i(x)}{dt} = -\begin{bmatrix} \left(a_{i1} - \sum_{k=1}^m b_{ik}k_{k1}\right)x_1 + \left(a_{i2} - \sum_{k=1}^m b_{ik}k_{k2}\right)x_2 + \\ +,,,+\left(a_{in} - \sum_{k=1}^m b_{ik}k_{kn}\right)x_n \end{bmatrix}^2, \quad (8)$$

$$i = 1,2,...,n$$

From the expressions (8) that the total time derivative of the vector-Lyapunov $V_i(x)$ functions in the performance of the initial assumptions resulting from the geometric interpretation of a theorem A.M. Lyapunov will be negative sign function. This means that the conditions for asymptotic stability of the system will always be performed (4).

Now, using components of the gradient vector we will restore components of the vector Lyapunov functions:

$$V_i(x_1,x_2,...,x_n) = -\left(a_{i1} - \sum_{k=1}^m b_{ik}k_{k1}\right)x_1^2 - \left(a_{i2} - \sum_{k=1}^m b_{ik}k_{k2}\right)x_2^2 -, \\ -...,-\left(a_{in} - \sum_{k=1}^m b_{ik}k_{kn}\right)x_n^2, \\ i = 1,2,...,n$$

The positive definiteness of all components of the vector Lyapunov function will be expressed by

$$-\left(a_{ij} - \sum_{k=1}^m b_{ik}k_{kj}\right) > 0, i = 1,2,...,n,\ j = 1,2,..,n \quad (9)$$

This condition characterized superstability of transposed matrix of a closed system [4].

IV. The Robust Stability Condition and Radius of the Robustness

Let us investigate the robust stability of the vector-Lyapunov functions. Then let us transform the condition of robust stability of the components of the vector Lyapunov function. For this, we can turn to a parametric family of coefficients the vector-Lyapunov functions, such as the interval family, defined as [4]:

$$d_{ij} = d_{ij}^0 + \Delta_{ij}, \left|\Delta_{ij}\right| \le \gamma\ m_{ij}, i, j = 1,2,...,n$$

where the nominal rate

$$d_{ij}^0 = -\left(a_{ij}^0 - \sum_{k=1}^m b_{ik}^0 k_{kj}^0\right)$$

corresponds to a positive-definite Lyapunov functions, i.e.

$$\sigma(D_0) = \min_i \min_j -\left(a_{ij}^0 - \sum_{k=1}^m b_{ik}^0 k_{kj}^0\right) > 0$$

Now, we require that the positivity condition coefficients stored for all functions of the family:

$$-\left(a_{ij}^0-\sum_{k=1}^{m}b_{ik}^0k_{kj}^0\right)+\Delta_{ij}>0, i=1,2,...,n;\ j=1,2,...,n$$

Clearly, this inequality holds for all admissible Δ_{ij} if and only if

$$-\left(a_{ij}^0-\sum_{k=1}^{m}b_{ik}^0k_{kj}^0\right)+\gamma\, m_{ij}>0,$$

$$i=1,2,...,n;\ j=1,2,...,n$$

i.e. when

$$\gamma<\gamma^*=\min_i\min_j\frac{-\left(a_{ij}^0-\sum_{k=1}^{m}b_{ik}^0k_{kj}^0\right)}{m_{ij}} \qquad (10)$$

In particular, if $m_{ij}=1$ (scale factors of a member of Lyapunov functions are the same), then

$$\gamma^*=\sigma(D_0) \qquad (11)$$

Thus, the stability radius of interval family of positive definite functions is the smallest value of the coefficients of the vector Lyapunov functions. As an example, we consider the system described in state space. Let *n = 2, m = 1*, i.e.,

$$\dot{x}=Ax+Bu,\ A=\begin{Vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{Vmatrix},$$

$$B=b=\begin{Vmatrix}b_1\\ b_2\end{Vmatrix},\ u=-Kx,\quad K=k=\begin{Vmatrix}k_1 & k_2\end{Vmatrix}$$

$$\dot{x}_1=(a_{11}-b_1k_1)x_1-(a_{12}-b_1k_2)x_2$$
$$\dot{x}_2=(a_{21}-b_2k_1)x_1-(a_{22}-b_2k_2)x_2 .$$

Then

$$G=A+BK=\begin{Vmatrix}a_{11}-b_1k_1 & -(a_{12}-b_1k_2)\\ a_{21}-b_2k_1 & -(a_{22}-b_2k_2)\end{Vmatrix}$$

With inequality [4] characteristic equation has roots with negative real parts.

$$\begin{cases}a_{22}-b_2k_2-a_{11}+b_1k_1>0\\ (a_{12}-b_1k_2)(a_{21}-b_2k_1)-(a_{11}-b_1k_1)(a_{22}-b_2k_2)>0\end{cases}$$

We investigate the stability of the system using the idea of Lyapunov functions.

Let us investigate the components of the gradient vector components vector functions $V_1(x_1,x_2)$ and $V_2(x_1,x_2)$:

$$\frac{\partial V_1(x_1,x_2)}{\partial x_1}=-(a_{11}-b_1k_1)x_1,\ \frac{\partial V_1(x_1,x_2)}{\partial x_2}=+(a_{12}-b_1k_2)x_2$$
$$\frac{\partial V_2(x_1,x_2)}{\partial x_1}=-(a_{21}-b_2k_1)x_1,\ \frac{\partial V_2(x_1,x_2)}{\partial x_2}=+(a_{22}-b_2k_2)x_2$$

We discover the total time derivative of the Lyapunov function by the formula (8):

$$\frac{dV(x_1,x_2)}{dt}=-\left[(a_{11}-b_1k_1)x_1+(a_{12}-b_1k_2)x_2\right]^2-$$
$$-\left[(a_{21}-b_2k_2)x_1+(a_{22}-b_2k_2)x_2\right]^2<0$$

The next step - discovering vector Lyapunov functions

$$V_1(x_1,x_2)=-\frac{1}{2}(a_{11}-b_1k_1)x_1^2+\frac{1}{2}(a_{12}-b_1k_2)_2^2$$
$$V_2(x_1,x_2)=-\frac{1}{2}(a_{21}-b_2k_1)x_1^2+\frac{1}{2}(a_{22}-b_2k_2)x_2^2$$

Conditions for the stability of the system obtained in the form:

$$-(a_{11}-b_1k_1)>0,\ (a_{12}-b_1k_2)>0,$$
$$-(a_{21}-b_2k_1)>0,\ (a_{22}-b_2k_2)>0$$

and

$$-(a_{11}-b_1k_1)>(a_{21}-b_2k_1),$$
$$(a_{22}-b_2k_2)>-(a_{12}-b_1k_2)$$

From this we can get a system of inequalities

$$a_{22}-b_2k_2-a_{11}+b_1k_1>0$$
$$(a_{12}-b_1k_2)(a_{21}-b_2k_1)-(a_{11}-b_1k_1)(a_{22}-b_2k_2)>0$$

Thus, from (9) and (10) we can determine the radius of robust stability of a second order system, if system parameters are uncertain [7,8]:

$$\gamma^*=\min\begin{Bmatrix}-(a_{11}-b_1k_1),(a_{12}-b_1k_2),\\ -(a_{21}-b_2k_1),(a_{22}-b_2k_2)\end{Bmatrix}$$

Then, as an example, we define the following initial conditions and find conditions for the stability of the system, the radius and transients.

When the initial settings are follow:

$$A=\begin{Vmatrix}-11.6 & 45\\ 11.7 & 0.4\end{Vmatrix},B=\begin{Vmatrix}9.2\\ 7\end{Vmatrix},K=\begin{Vmatrix}2 & 0.001\end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 0.3860$).

The overall the transition process of the system shows on the Figure 1

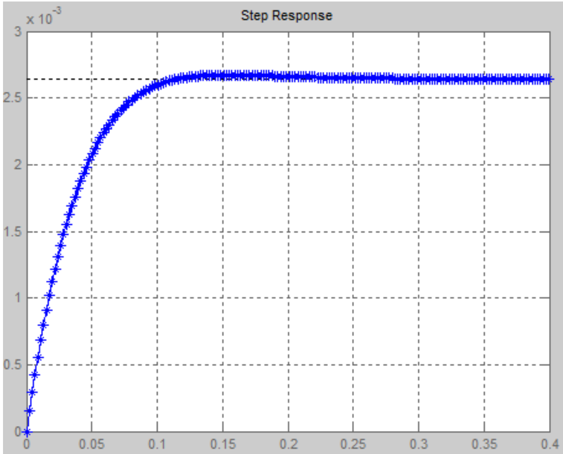

Fig. 1.The transition process, exp.1.

The second case, when the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 3 \\ 0.8 & 4 \end{Vmatrix}, B = \begin{Vmatrix} 3 \\ 17 \end{Vmatrix},$$

$$K = \begin{Vmatrix} 3.2 & -0.002 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 3.01$).

The overall the transition process of the system shows on the Figure 2.

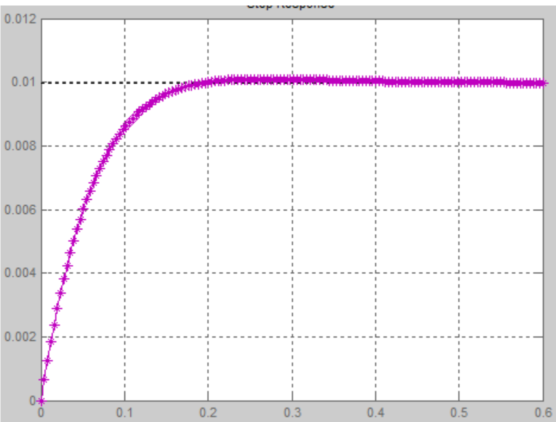

Fig. 2.The transition process, exp.2.

The third case, when the initial settings are follow:

$$A = \begin{Vmatrix} -15.6 & 45 \\ 11.7 & 1.4 \end{Vmatrix}, B = \begin{Vmatrix} 8.2 \\ 9.2 \end{Vmatrix},$$

$$K = \begin{Vmatrix} 8 & 0.02 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 1.032$).

The overall the transition process of the system shows on the Figure 3.

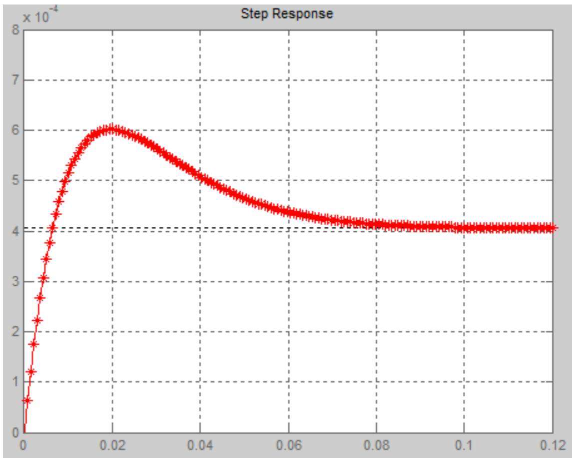

Fig. 3.The transition process, exp.3.

For 4-d case, when the initial settings are follow:

$$A = \begin{Vmatrix} -11.6 & 45 \\ 11.7 & 1.4 \end{Vmatrix}, B = \begin{Vmatrix} 8.2 \\ 9.2 \end{Vmatrix},$$

$$K = \begin{Vmatrix} 8 & 0.02 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 1.032$).

The overall the transition process of the system shows on the Figure 4.

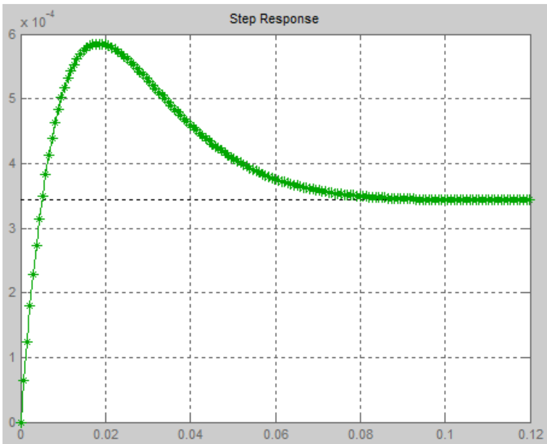

Fig. 4.The transition process, exp.4.

For 5-d case, when the initial settings are follow:

$$A = \begin{Vmatrix} -21.6 & 45 \\ 11.7 & 1.4 \end{Vmatrix}, B = \begin{Vmatrix} 8.2 \\ 9.2 \end{Vmatrix},$$

$$K = \begin{Vmatrix} 8 & 0.02 \end{Vmatrix}$$

In this case, the radius will be equal ($\gamma^* = 1.032$). The overall the transition process of the system shows on the Figure 5.

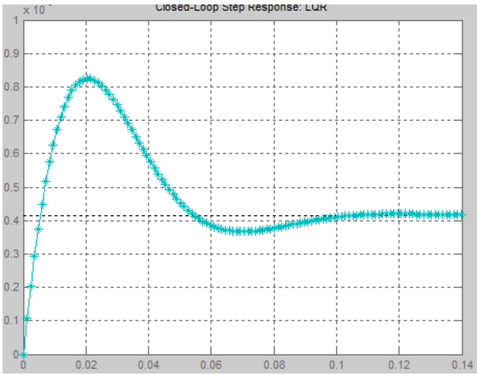

Fig. 5.The transition process, exp.5.

The complex analysis of examples we can see on Figure 6. For all the examples given initial values and the stability conditions of the system are executed.

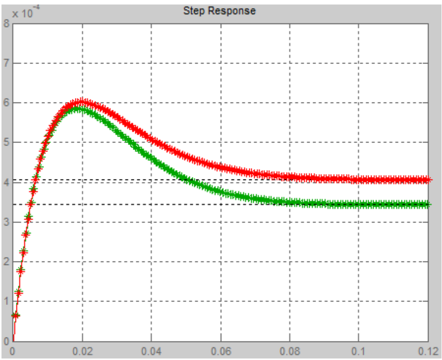

Fig. 6.The transition process, exp.3-4.

V. Conclusion

In our theory robust stability perform an important function in the theory of control of dynamic objects is [7,8]. The main point of robust stability study is to specify constraints on the change control system parameters that preserve stability. These limits are determined by the region of stability in an uncertain and are selected, i.e. changing parameters [9,10,11,12]. In this paper we propose an approach of the construction of a Lyapunov function in the form of a vector function in way that it is equal to the gradient of the components of the velocity vector (right side of the equation of state), but with the negative sign.

Study of the robust stability of the system is based on the idea of a direct method A.M. Lyapunov. The region of stability is obtained in the form of simple inequalities for uncertain parameters control object and selected regulator properties. A new theoretical method of robust stability is proposed for linear systems with uncertain valued parameters. This method is an extension of the notion of stability where the Lyapunov function is replaced by a geometric interpretation of the Lyapunov function with dependence on the uncertain parameters [11,12,13]. The radius of stability coefficients interval family of positive definite functions is equal to the smallest value of the coefficients of the vector Lyapunov functions. Theoretical results obtained in this paper are an important contribution to the theory of stability, to the theory of robust stability of linear control systems. Thus, for a wide class of systems, we believe the theory is sufficiently well developed that work can begin on developing efficient approach to aid control engineers in incorporating the parametric approach into their analysis and design toolboxes. The practical importance of these results should motivate new theoretical studies on typical application techniques, clarification area of the robust control and stability [13].

Finally, this is the main results that theoretical approaches represent the most promising direction. These studies are especially important for the designing more effective control systems.

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